Lesson 3: Karnaugh Maps

Computer Aided Digital Design
EE 3109
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Karnaugh Maps
- Boolean Algebra – no guarantee minimized expression
  - reduction-order dependent
  - still necessary to sanity check results
- Need other more systematic methods
  - Karnaugh Maps – graphical representation
  - Quine-McCluskey method – computer algorithm

Three Variable Karnaugh Maps – minterm positions

Three Variable Karnaugh Maps - example

Three Variable Karnaugh Maps - legal circles

Three Variable Karnaugh Maps - example
Three Variable Karnaugh Maps – wrap around

\[ Z = A'B'C' + A'B C' \]
\[ Z = A'C' \]

Three Variable Karnaugh Maps

\[ Z = A'B'C' + A'B C' + A B C' + A B'C' \]
\[ Z = C' \]

Three Variable Karnaugh Maps

\[ Z = A'C + A B + B'C \]
\[ \text{Proof of consensus term being redundant} \]

Sample Problem 1 – 5 min

Four Variable Karnaugh Maps – minterm expansions

\[ Z(a,b,c,d) = \sum m(0,1,8,9) \]
\[ Z = B'C' \]
Four Variable Karnaugh Maps – example 2

\[ Z = \overline{C} D + CD \]

\[ Z = \overline{D} \]

Four Variable Karnaugh Maps – example 3

\[ Z = C' \]

\[ \sum m(0, 1, 4, 5, 8, 9, 12, 13) \]

Four Variable Karnaugh Maps – example 4

\[ Z = C'D' + AB + ABC' \]

Sample Problem 2 – 5 min

\[ Z = C'D' + A'B' + AC' + B'C \]

Hazards

Hazards - fixes
### BCD – clock example

**BCD – Binary Coded Decimal**

1 2 : 4 5

0001 0010 : 0100 0101

Values 1010 through 1111 not legal values

### Seven Segment Display

**BCD to Seven Segment Decoder**

- 4-bits input
- 7-bits output
- Unused bits
  - Input greater than 9?
  - Use last digit of your student ID
- Minimize the number of gates
- Calculate the fan-in of circuit

<table>
<thead>
<tr>
<th>BCD</th>
<th>a</th>
<th>b</th>
<th>c</th>
<th>d</th>
<th>e</th>
<th>f</th>
<th>g</th>
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